A Fast Remesh-free Mesh Deformation Method Based on Radial Basis Function Interpolation and Its Application to Optimal Design of Electromagnetic Devices

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Optimal shape design of electromagnetic (EM) devices using finite element parameter sweeping analysis is becoming a routine procedure today. In the optimal design process, different meshes for different device shapes are required for subsequent finite element computations in order to obtain the objective function values from the field solution. Traditional full or partial remeshing methods usually consume excessive computing time when generating these finite element meshes, especially for three-dimensional (3-D) problems. The proposed remesh-free method being reported is based on a radial basis function (RBF) interpolation technique, which can save the mesh-regeneration time and greatly accelerate the optimal design process. The proposed method needs a fine finite element mesh which can be generated by any external meshing software. An algorithm to find the boundary nodes located on the geometry outlines or material interfaces is proposed to prescribe the source displacement vectors there. A RBF interpolation function for the boundary nodal motion is then being solved and this process needs to be done once and only once. If the geometry is updated for new design parameters, the displacement vectors for all the nodes can be interpolated from the reconstructed RBF and hence the proposed mesh deformation method is remesh-free. The method can be applied to both two-dimensional (2-D) and 3-D problems. 2-D and 3-D numerical examples are given to showcase the effectiveness of the proposed method.

***Index Terms*—Finite element, mesh deformation, optimal shape design, three-dimensional magnetic field.**

I. Introduction

TO IMPROVE the performance of electromagnetic (EM) devices in the process of designing new products, geometric sizing or shape optimization is very important in practice. In the optimization process, because there are frequent variations in the design parameters, the computational mesh has to be generated repeatedly before one can proceed to the finite element (FE) computation. Thus it is highly desirable to reduce the computing time required in mesh regeneration if there is no need to fully remesh the solution domain.

Several methods are available to update the mesh associated with the new design parameters from the previous mesh. One method is to partially remesh the domain by solving the equations of elasticity [1] or the Laplace equation when calculating the new coordinates of the current mesh [2]. However this type of partial remeshing method is rather time consuming hitherto and not very robust. It may even generate folding elements if there are large shape modifications [3].

To overcome the drawbacks of the above mentioned methods when constructing the meshes for new geometric parameters, a parameterized meshing technique which is remesh-free has been proposed [4-7] for fast two-dimensional (2-D) mesh deformation. It utilizes a novel vertex data structure containing the design parameters and combines mesh generation and the refinement process that require a full grasp of the whole meshing programs. In practice, because different engineers prefer different meshing software packages to make the mesh, it is highly useful to directly deform the user-given finite element mesh, which can be generated by any external meshing software, especially for three-dimensional (3-D) optimal design problems.

In this work, a remesh-free method is proposed and which is based on a radial basis function (RBF) interpolation technique, which can save the mesh-regeneration time and greatly accelerate the optimal design process. The idea has been used in the optimal shape design of airfoils and proven to be effective [8]. However it has not been applied to the optimal design of electromagnetic devices, to the best of our knowledge, such as electric motors where multiple materials are presented and the problem domain is overly complicated. In this work, the method formerly developed is generalized to both 2-D and 3-D complicated geometric domains containing multiple materials.

The proposed method needs a user-given fine finite element mesh which can be generated by any external meshing software. An algorithm to find the mesh nodes located on the geometry outlines or material interfaces is proposed to prescribe the source displacement vectors for these boundary nodes. The solution of the RBF interpolation process needs to be done only once. Once the geometry is updated, the displacement vectors for all the interior nodes can be interpolated from the reconstructed RBF and hence the proposed mesh deformation method is remesh-free.

The method can be applied to both 2-D and 3-D problems. Numerical examples will be given to showcase the effectiveness of the proposed method. The contribution of this work is to provide a novel fast and robust mesh deformation method to reduce the time needed for generating the meshes for different design parameters to accelerate the optimal shape design process. In the full paper engineering optimal design problems will be presented to further showcase its effectiveness.

II. Mesh Deformation Method Based on RBF

In this work a remesh-free mesh deformation method based on RBF interpolation is proposed for fast and robust mesh deformation of both 2-D and 3-D geometries. The proposed method requires only one set of fine FE computational mesh. An algorithm of extracting the nodes located on the geometry outlines or material interfaces in the mesh is proposed. Then one can prescribe the desired displacement vectors for these boundary nodes, which are then being interpolated by the

RBFs for which multiple choices of the basis functions are available [8]. After solving the RBF interpolated function, which needs to be done only once, the displacement vectors of all the other nodes in the mesh can then be calculated from this reconstructed function.

A. *Finding the Nodes on Material Interfaces*

For any mesh generator, the generated mesh will at least contain the information of the coordinates of each node, the material information of each element and element-node connectivity. After reading the user-given mesh file into the computer memory, one can construct and store an integer array of the elements that are connected to each node in the mesh. The node is a boundary node on the material interfaces if and only if its neighboring elements are not with the same material markers. By using this algorithm it is very convenient to mark the nodes on the material interfaces, which are also called boundary nodes. Then one can prescribe the displacement vectors for these boundary nodes when the design parameters are changed. By reconstructing a function based on the RBF over the whole domain, all the nodal displacement vectors can be obtained.

B. *Mesh Deformation Examples by the Proposed Method*

Two mesh deformation examples are given to showcase the effectiveness of the proposed method.

The first one is a 2-D example on shape optimization of a direct current (DC) magnetic polewhich is required to generate its magnetic fields as uniformly as possible in the airgap. For illustration, the user-given finite element mesh is shown in Fig. 1(a). The extracted boundary nodes and the boundary mesh of the device is given in Fig. 1(b). The deformed boundary mesh is shown in Fig. 1(c). Using the proposed RBF interpolation technique, the displacement vectors are shown in Fig. 1(d), where one can observe that the modification of the geometric outlines are propagated to the whole domain smoothly.

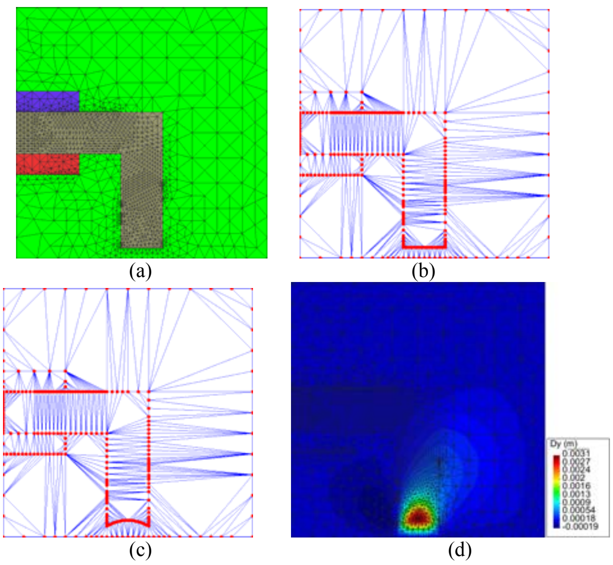

Fig. 1 (a) User-given finite element mesh. (b) Initial boundary mesh extracted from the fine mesh. (c) Updated boundary mesh for new design parameters. (d) Deformed finite element mesh and contour plot of the absolute values of all the nodal displacement vectors after mesh deformation using RBF interpolation technique.

In the second example, the armature of the device as shown in Fig. 2 will be rotated at the center of a fixed point around the *x*-axis counterclockwisely. The initial mesh with $\alpha = 0^0$ having 127,967 tetrahedral elements and 22,137 nodes is shown in Fig. 3(a). The deformed mesh with $\alpha = 5^0$ is given in Fig. 3 (b). It is noted that it takes about 85 seconds by the mesh generator to generate the conforming mesh shown in Fig. 3(a). However it only costs 0.2 second to deform the mesh using the proposed RBF interpolation technique. Hence the mesh regeneration time is almost totally saved using the remesh-free RBF deformation technique, when compared with that of the traditional brute force sweeping method.

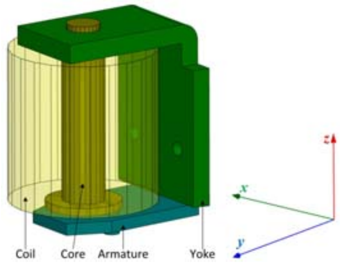

Fig. 2. Definition of the problem.

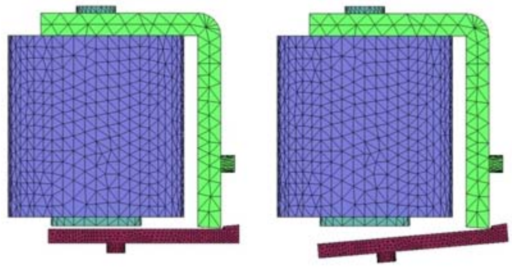

Fig. 3. (a) The initial mesh when $\alpha = 0^0$. (b) The deformed mesh using the remesh-free mesh deformation method with $\alpha = 5^0$.

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